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16. Abstract

Systematic errors arising in photometry of planets are discussed. These errors are found to be caused by diffraction by the lens mount, blurring of the image due to atmospheric turbulence, and photographic irradiation in the case of photographic photometry.

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CERTAIN SYSTEMATIC ERRORS ARISING IN THE PHOTOMETRY OF PLANETARY DISCS

V. N. Dudinov

Just like any other measurement process, photometry of planets entails random and systematic errors. At the present time, in measurements of the brightness distribution over the planet disc an accuracy may be readily reached such that the internal convergence of the results will be on the order of 1%. This comparatively high photometry accuracy made it possible for M. Minnaert [1] and later V. I. Yezerskiy [2] to use the Helmholtz reciprocity principle to study the homogeneity of planetary atmospheres. However, the internal convergence of the results in itself cannot be a sufficiently reliable criterion for the data obtained. Therefore, we shall establish certain systematic errors which may be included not only in the photometric catalogs of different authors, but also the results of comparing them. / 77*

In the photometry of planets, systematic errors, which are to a certain extent inherent to all observations, may be caused by diffraction on the lens mount, by blurring of the image due to atmospheric turbulence, and also by photographic irradiation in the case of photographic photometry.

Many authors have studied these errors, including G. Struv'ye, [7], V. G. Fesenkov [8], N. P. Barabashov [9], I. A. Parshin [10], V. N. Lebedinets [11] and others. However, these studies

* Numbers in margin indicate pagination in original foreign text.

practically always were reduced to determining the zone near the disc edge, where the systematic error in measuring the brightness could surpass the random photometry errors.

We set ourselves the problem of at least making a qualitative analysis of systematic errors over the entire planetary disc.

Just as in [3, 4, 5], we shall approximately assume that the brightness distribution measured over the planet disc is a linear transformation of the real distribution with a brightness nucleus, i.e.,

$$F(x, y) = \iint_G f(x', y') g(x - x', y - y')\, dx' dy',$$

where $F(x,y)$ - is the measured brightness distribution over the planet disc;

$f(x,y)$ - true brightness distribution;

$g(x,y)$ - instrument function of the device; in this case, this is the measured brightness distribution in terms of the stellar image.

Integration is only performed over the region filled with the geometric dimensions of the luminous planet disc.

1. Errors in Determining the Parameter q of the Planet

In the studies [3] and [4], the authors point to the systematic error which arises in a determination of the parameter q (planet smoothness factor), which is caused by blurring of the image due to scattering of light in the Earth's atmosphere, /78 so that q obtained by different authors is always too large. Let us discuss this question in greater detail.

It was shown in [5] that blurring of the planet disc may be primarily regarded as a limitation of the higher spatial frequencies in the image. It may be seen from the results obtained at the Main Astronomical Observatory of the USSR Academy of Sciences [4], that the brightness distribution over the stellar image may be satisfactorily approximated by a Gaussian curve $g(r) \sim e^{-\frac{r^2}{2\sigma^2}}$ with σ on the order of 1".5-2".

In this case, the function which limits the higher spatial frequencies and which is a Fourier transformation g(r) is also described by the Gaussian curve

$$g(\omega) \sim e^{-\frac{\sigma^2 \omega^2}{2}}.$$

Just as in [5], all of the subsequent calculations will be performed in the one-dimensional approximation. By analogy with radio physics, we shall call $g(\omega)$ the frequency characteristic of a linear device, and the region in which $g(\omega)$ is defined — its pass band.

Let us represent the brightness distribution over the planet disc during the opposition in the form of a Sytinskaya formula for rough surfaces

$$B(q, x) = B_0 (1 - x^2)^{q/2},$$

where B_0 — is the brightness in the center of the disc;
q — smoothness factor $(0 \leqslant q \leqslant 1)$;
x — coordinate determined from the center of the disc.

We may find the Fourier transformation B(x) in the form

$$\Phi(q, \omega) = B_0 2^{q/2} \Gamma\left(1 + \frac{q}{2}\right) J_{\frac{q+1}{2}}(\omega)\, \omega^{-\frac{q+1}{2}}.$$

It may be readily seen that the main portion of the energy of any of the functions $\Phi(q, \omega)$, т. е. $\int_{-\infty}^{+\infty} |\Phi(q, \omega)|^2 d\omega$ is concentrated in the interval ω included between the first zeros of the function $J_{\frac{q+1}{2}}(\omega)$, i.e., approximately in the band $|\omega| < 3$.

Taking into account the Parseval equation, we may write

$$\int_{-\infty}^{+\infty} |B(q,x)|^2 dx = \int_{-\infty}^{+\infty} |\Phi(q,\omega)|^2 d\omega \approx \int_{-3}^{+3} |\Phi(q,\omega)|^2 d\omega = \int_{-\infty}^{+\infty} |B^*(q,x')|^2 dx'.$$

Here B (q,x) and B*(q,x') are the true and measured brightness distribution over the planet disc.

In view of the small difference between the energy of the measured and true distribution function, even at $\sigma \sim 0{,}3 \div 0{,}4\ (\omega < 3)$, the difference between the measured brightness distributions over the planet disc for different q remains almost the same as for the true distributions.

If it is known beforehand that the brightness distribution over the planet disc satisfies the Sytinskaya formula, then to obtain the true brightness distribution it is only necessary to find a method which may be used to compare the true B (q,x) to each measured distribution B* (q,x'). Thus, the measurement error must not exceed a certain value which characterizes the difference of the distributions for different q. However, in practice the situation is much more complex.

By way of an example, in the case of two extreme values of the parameter q, let us examine the function $\Phi(q,\omega)$:

$$\Phi_0(\omega) = B_0 \sqrt{\frac{2}{\pi}} \frac{\sin\omega}{\omega} \text{ for } q = 0;$$

$$\Phi_1(\omega) = B_0 \sqrt{\frac{\pi}{2}} \frac{I_1(\omega)}{\varphi} \text{ for } q = 1.$$ /79

It is found that the behavior of the functions $I_1(\omega)$ and $\frac{1}{2}\sin\omega$ is the same for small values of ω, and close to zero $\Phi_1(\omega)$ and $\Phi_0(\omega)$ differ only by the constant factor

$$\Phi_1(\omega) = \frac{\pi}{4}\Phi_0(\omega).$$

Asymptotically at $\omega \to \infty$ $\Phi_1(\omega)$ strives to zero as $\omega^{-3/2}$ and $\Phi_0(\omega)$ — as ω^{-1}.

It may be shown that the mean square deviation $\frac{\pi}{4}\Phi_0(\omega)$ from $\Phi_1(\omega)$

$$\Delta = \sqrt{\frac{\int_{-\omega_0}^{\omega_0}\left[\Phi_1(\omega) - \frac{\pi}{4}\Phi_0(\omega)\right]^2 d\omega}{\int_{-\omega_0}^{\omega_0}[\Phi_1(\omega)]^2 d\omega}} = \sqrt{\frac{\int_{-\infty}^{+\infty}\left[B_1(\omega) - \frac{\pi}{4}B_0^*(x)\right]^2 d\omega}{\int_{-\infty}^{+\infty}[B_1^*(x)]^2 dx}}$$

within the limits of the band $|\omega| < 1$ does not exceed 1%, and within the limits $|\omega| < 3$ does not exceed 12%. Only in the limits $|\omega| < 7$ does it reach 25%. Apparently, for any other values of the parameter q, this difference will be smaller.

Thus, two brightness distribution functions which are characterized by a differing value of the parameter q at sufficiently large σ will differ only by a constant factor. However, the difference in the constant factor is not important, since it is included in the brightness distribution over the disc as the unknown albedo of the planet. We should note that a similar phenomenon may occur at phases differing from zero.

In these cases, the most reliable information regarding the brightness distribution over the disc may be obtained from measuring the phase function, assuming that the law governing the brightness change with phase is known. Strictly speaking, this principle was used to obtain the value of the parameter q for Mars by N. P. Barabashov, Yu. V. Aleksandrov and V. I. Garazha in the study [6].

With respect to the measured brightness distribution function over the disc, at sufficiently large σ for any q it will have the smallest deviation from the true function which has the narrowest spectrum. In the case of the opposition, this function is the Lambert function for which q = 1.

If we do not take into account the systematic distortions caused by blurring of the image and assume that the measured brightness distribution is the real distribution, then, as was shown in [3, 4], the value obtained for the parameter q will be too large. In actuality, in the case of a law close to the Lambert law, even for σ which is not too large, the random error in determining the zero brightness (background on the photographic plate) may lead to a decrease and to an increase in this parameter. Thus, the statement made in [3, 4] regarding the systematic exaggeration of the parameter q loses any meaning at q which are close to 1, since in this case the error of determining q has a more random nature. /80

2. Certain Errors in Measuring Brightness Distribution over the Planet Disc at an Arbitrary Phase

The brightness distribution over the planet disc is a certain function of two coordinates f(x,y) within the region bounded by the lines $x=\sqrt{1-y^2}$ and $x=\cos\alpha\sqrt{1-y^2}$, i.e., the geometric dimensions of the luminous disc. Outside of this region, the function f(x,y) identically equals zero.

Since the brightness distribution over the stellar image may be readily approximated by the function $q(x, y) \sim e^{-\frac{x^2+y^2}{2\sigma^2}}$, the measured brightness distribution over the planet may be represented, at least approximately, in the form

$$F(x, y)=\frac{1}{2\pi\sigma^2}\iint\limits_{G(x, y)} f(x-x', y-y')e^{-\frac{x'^2+y'^2}{2\sigma^2}}dx'dy', \tag{1}$$

where the integration is performed only over the region of defining f(x,y).

It may be assumed that the function f(x,y) is continuous within the region G. The measured function F(x,y) is continuous on the entire plane in view of the continuity of the kernel of the given integral equation [14].

In measuring the brightness distribution over the planet disc, we are most interested in the distribution along the equator of intensity. As will be shown below, in this case f(x,y) may be approximately assumed to depend only on one coordinate, i.e., it is assumed that the planet over the y axis extends to infinity and its brightness does not depend on y. This assumption makes it possible to comparatively easily perform a numerical determination of the deviations between the true and measured brightness distribution over the planet disc for any specific distribution. The results of calculations in the one-dimensional case are given in Figures 1-6 for certain specific distributions.

As follows from the graphs presented, the error in determining the planet disc center may be substantial. The error may be more systematic than it is random — the center of the disc found from the observations will most frequently be displaced towards the limb (compare with [16]. In addition, in the case of an arbitrary law governing the brightness distribution, displacements of the brightness maximum must be expected, as well as disturbances of the reciprocity principle, since the distorting influence of the atmosphere will be greater on the limb.

Let us now determine the error caused by replacing a two-dimensional brightness distribution by a one-dimensional distribution.

Since the integrand in (1) rapidly strives to zero outside of the circle $x^2+y^2=\sigma^2$, for small σ we may use the expansion f(x-x', y-y') in Taylor series close to a point with the coordinates (x,y), and we may substitute only certain expansion

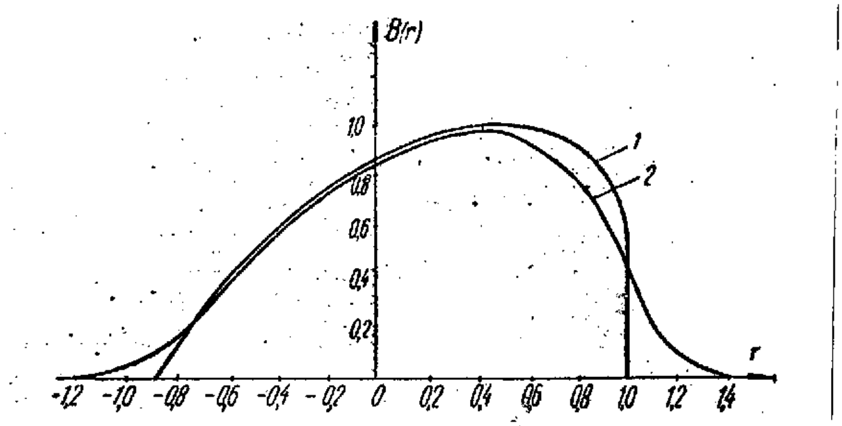

Figure 1. Brightness distribution.

1- undistorted at $\alpha = 30°, k = 1$ (Lambert reflection law);
2- distorted at $\sigma = 0,2$.

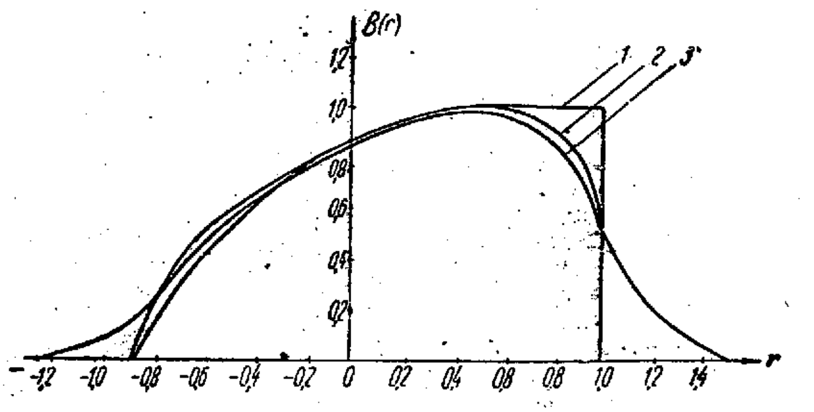

Figure 2. Brightness distribution.

1- undistorted at $\alpha = 30°, k = 0$ (reflection from extremely pitted surface); 2- undistorted at $\alpha = 30°, k = 1$; 3 — distorted for $\alpha = 30°, k = 0, \sigma = 0,2$.

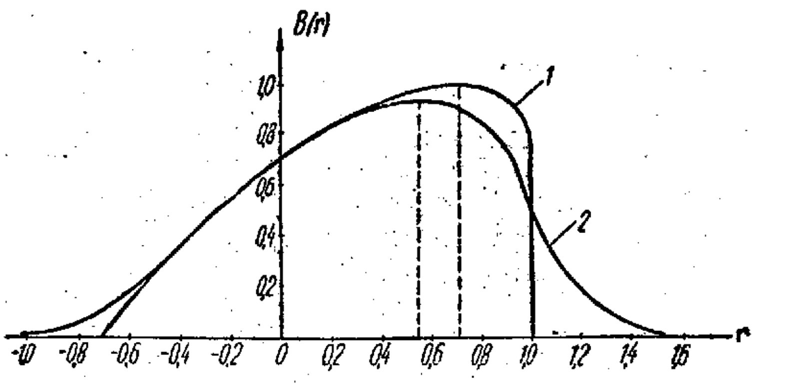

Figure 3. Brightness distribution.

1- undistorted at $\alpha = 45°, k = 1$; 2 — distorted at $\sigma = 0,2$.

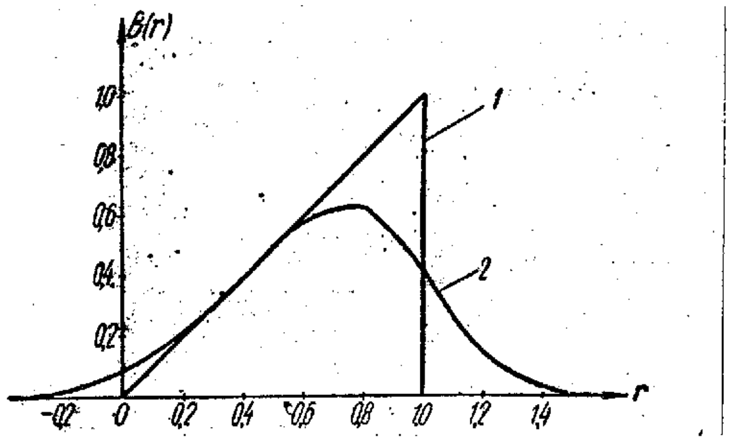

Figure 4. Brightness distribution.
1- undistorted at $\alpha = 90°,\ 0 < k < 1$; 2 — distorted at $\sigma = 0{,}2$.

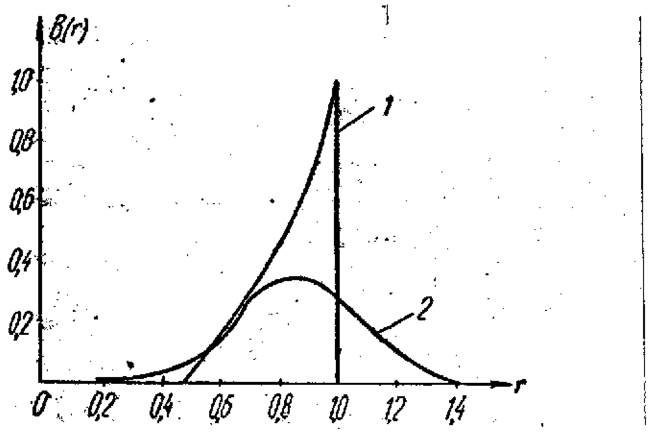

Figure 5. Brightness distribution.
1- undistorted at $\alpha = 120°\ 0 < k < 1$; 2 — distorted at $\sigma = 0{,}2$.

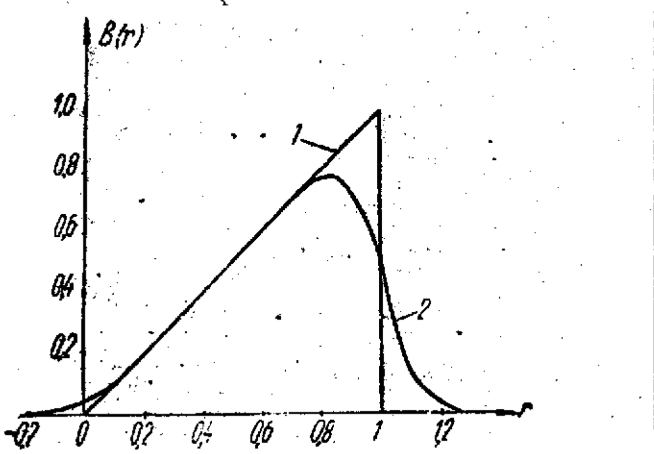

Figure 6. Brightness distribution.
1- undistorted at $\alpha = 90°\ 0 < k < 1$; 2 — distorted at $\sigma = 0{,}1$.

terms in (1). Then for the equator of intensity

$$
\begin{aligned}
F(x,\ 0) = {} & \frac{f(x,0)}{2\pi\sigma^2}\int\limits_{-(a+x)}^{(b-x)} e^{-\frac{x'^2}{2\sigma^2}}\,dx' \int\limits_{-\varphi(x-x')}^{\varphi(x-x')} e^{-\frac{y'^2}{2\sigma^2}}\,dy' + \\
& + \frac{f'_x(x,0)}{2\pi\sigma^2}\int\limits_{-(a+x)}^{(b-x)} x' e^{\frac{x'^2}{2\sigma^2}}\,dx' \int\limits_{-\varphi(x-x')}^{\varphi(x-x')} e^{-\frac{y'^2}{2\sigma^2}}\,dy' + \\
& + \frac{f''_{xx}(x,0)}{4\pi\sigma^2}\int\limits_{-(a+x)}^{(b-x)} x'^2 e^{-\frac{x'^2}{2\sigma^2}}\,dx' \int\limits_{-\varphi(x-x')}^{\varphi(x-x')} e^{-\frac{y'^2}{2\sigma^2}}\,dy' + \\
& + \frac{f''_{yy}(x,0)}{4\pi\sigma^2}\int\limits_{-(a+x)}^{(b-x)} e^{-\frac{x'^2}{2\sigma^2}}\,dx' \int\limits_{-\varphi(x-x')}^{\varphi(x-x')} y'^2 e^{-\frac{y'^2}{2\sigma^2}}\,dy' + \ldots
\end{aligned}
\tag{2}
$$

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Here a and b are the boundaries of the luminus disc along the x axis; $\varphi(x)$ determines the boundary of the region in which f(x,y) is specified.

The absence of terms with the first derivative with respect to y and a mixed derivative in (2) may be explained by two reasons. In the first place, the points of the equator of intensity for any section parallel to the y axis may be assumed to be points of the extremum, where $f'_y(x,\ 0) = 0$, and this means $f''_{xy}(x,\ 0) = 0$. In the second place, integration over y' is carried out within symmetrical limits and the integral $\int\limits_{-\varphi(x-x')}^{\varphi(x-x')} y'^3 e^{-\frac{y'^2}{2\sigma^2}}dy'$ of the antisymmetric function equals zero. We should note that both of these conditions are not satisfied for sections parallel to the equator of intensity.

If we set $\frac{1}{\sqrt{2\pi\sigma^2}}\int\limits_{-\varphi(x-x')}^{\varphi(x-x')} e^{-\frac{y'^2}{2\sigma^2}}dy' = 1$, which is valid for small σ along the equator of intensity with the exception of the edge of the disc itself up to $x \sim \sqrt{1-\sigma^2}$, then

$$
F(x,\ 0) \approx F(x) + \frac{\sigma^2}{2} f''_{yy}(x,\ 0) \cdot \frac{1}{\sqrt{2\pi\sigma^2}} \int\limits_{-(a+x)}^{b-x} e^{-\frac{x'^2}{2\sigma^2}}dx' \tag{3}
$$

Formula (3) may be regarded as an asymptotic expression for (1) for small σ. It may be readily seen that the greatest errors of this asymptotic behavior will occur in the region with a width on the order of σ to both sides of the disc edge.

In view of the fact that

$$\frac{1}{\sqrt{2\pi\sigma^2}}\int_{-(a+x)}^{b-x} e^{-\frac{x'^2}{2\sigma^2}}dx'$$

does not exceed unity, for the equator of intensity we have the following estimate of the error caused by replacing the two-dimensional distribution of brightness by a one-dimensional distribution

$$\delta = F(x,0) - F(x) \leqslant \frac{\sigma^2}{2} f''_{yy}(x, 0). \tag{4}$$

This estimate is valid everywhere with the exception of the edge zone of the planet.

For points of the disc, located at a distance greater than σ from the edge, the function F(x,y) may, with the same degree of accuracy as (3), be represented in the form

$$F(x, y) = f(x, y) + \frac{\sigma^2}{2}\Delta f(x, y), \tag{5}$$

where

$$\Delta f(x, y) = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}.$$

Actually, for all points lying within the disc, with the exception of the edge zone with the width σ, the integration limits will be assumed to be infinite, as we assumed previously when integrating over y. Then the coefficient for (f(x,y) changes to unity, for $f'_x(x,y)$, $f'_y(x, y)$ and $f''_{xy}(x,y)$ — to zero, in view of the symmetry of the integration limits, and the coefficients equal $\frac{\sigma^2}{2}$ for $f''_{xx}(x, y)$ and $f''_{yy}(x, y)$. /82

We should note that both approximations are only obtained under the assumption of symmetry and smallness of the region in which the kernel of the integral equation (1) is specified, and the form of the kernel only determines the coefficient before the Laplace coefficient in (5).

To explain the systematic deviations between the true and measured brightness distribution over the planet disc, it is natural to limit all the possible distributions by a comparatively simple law which depends on the minimum amount of parameters and which satisfies a wider class of different distributions. All of the possible brightness distributions can naturally only be limited by distributions which satisfy the photometric homogeneity of the planet.

For this purpose, it is very advantageous to use a linear combination of two laws — the Lambert reflection law and the reflection law obtained by Akimov [12].

The Akimov formula was obtained under the assumption of an extremely pitted surface [13] and was found experimentally, just like the law governing reflection from the Moon. As is known, the Lambert law corresponds to smooth surfaces with a very large albedo.

We are only interested in relative brightness distribution over the planet disc in case of an arbitrary phase. There is a basis for assuming that a linear combination of these two brightness distributions satisfies, with a sufficient degree of accuracy, any arbitrarily pitted surface.

If we place the origin at the center of the disc and assume that the radius of the disc is 1, then the brightness distribution over the entire luminous disc for a Lambert surface will have the form

$$B(x,\ y,\ \alpha) = B_0\,[\cos\alpha\sqrt{1-x^2-y^2} + \sin\alpha x] = B_0 f_1(x,\ y,\ \alpha).$$

1. The function describing the brightness distribution over the disc for an extremely pitted surface, and the subsolar meridian, i.e., the region defined by the lines $x=\sqrt{1-y^2}$ and $x=\sin\alpha\sqrt{1-y^2}$

$$f_2(x,\ y,\ \alpha) = 1.$$

2. Between the subsolar meridian and the central meridian, i.e., in the region defined by the lines $x=\sin\alpha\sqrt{1-y^2}$ and $x=0$

$$f_2(x,\ y,\ \alpha) = [x\sin\alpha + \cos\alpha\sqrt{1-x^2-y^2}]\ \frac{1}{\sqrt{1-y^2}}$$

3. Between the central meridian and the terminator, i.e., in the region defined by the lines x = 0 and

$$x = -\cos\alpha\sqrt{1-y^2}$$
$$f^2(x,\ y,\ \alpha) = \frac{x\sin\alpha + \cos\alpha\sqrt{1-x^2-y^2}}{\sqrt{1-x^2-y^2}}$$

On the entire remaining plane, the functions $f_1(x,\ y,\ \alpha)$ and $f_2(x,\ y,\ \alpha)$ identically equal zero.

Following Minnaert, let us now formulate the reciprocity /83
principle for surfaces which are homogeneous in terms of their reflecting properties in the following form. Let us assume that there are two points on a homogeneous surface

$$a_1(i_1,\ \varepsilon_1)\ \text{и}\ a_2(i_2,\ \varepsilon_2),$$

where i — is the angle of incidence,

ε — reflection angle. Then, when the condition of symmetry is satisfied $i_1 = \varepsilon_2$ and $i_2 = \varepsilon_1$, the brightnesses of these points obey the following relationship.

$$\frac{B_1(i_1, \varepsilon_1)}{B_2(i_2, \varepsilon_2)} = \frac{\cos i_1}{\cos i_2} = \frac{\cos \varepsilon_2}{\cos \varepsilon_1}. \tag{6}$$

As may be readily seen, this principle is a direct corollary of the Helmholtz reciprocity principle. Both reflection laws satisfy the reciprocity principle and their linear combination satisfies it.

Thus, the brightness distribution over the disc for an arbitrarily pitted surface may be represented in the form

$$f(x, y, \alpha) = B_0(\alpha)\,[kf_1(x, y, \alpha) + (1-k)\,f_2(x, y, \alpha)]. \tag{7}$$

Here B_0 — is the brightness at a subsolar point which, generally speaking, depends on d;

k — porosity coefficient which in a certain way characterizes the surface porosity.

The parameter k determined in this way lies within $0 \leqslant k \leqslant 1$. In order to connect this parameter with the Sytinskaya smoothness factor q, we may find from (7) the brightness distribution when the phase equals zero.

If we plot the longitude λ along the abscissa axis, then we have the following from (7)

$$B(\lambda) = B_0\,(k\cos\lambda + 1 - k) = B_0\left((1 - 2k\sin^2\frac{\lambda}{2}\right).$$

For small λ, from the Sytinskaya formula we have

$$B(\lambda) = B_0\cos q\lambda \approx B_0\left(1 - 2q\sin^2\frac{\lambda}{2}\cos^2\frac{\lambda}{2} + \dots\right).$$

Thus, $k \approx q\cos^2\frac{\lambda}{2}$.

The brightness distribution over the planet disc which satisfies (7), even if for the same reason that formula (7) and the Sytinskaya formula depend only on one parameter, is a very cumbersome approximation which describes the reflection of light from an arbitrarily pitted surface. The relationship between the parameters, which is indeterminate over the entire disc, only points to the nonuniqueness of the representation by means of one parameter of the entire set of laws governing reflection from arbitrarily pitted surfaces.

The parameters k and q differ essentially only on the disc edge. Formula (7) does not give a zero brightness on the disc edge, in contrast to the Sytinskaya formula. This suggests the use of this formula, not only to describe the law governing reflection from solid surfaces, but also from planets having an atmosphere with a small optical thickness. It is true that in this case it is impossible to guarantee a constant parameter k at different phases.

Thus, we may write a law governing the brightness distribution over the disc which satisfies a certain wide class of different brightness distributions for any fixed phase angle

$$f(x, y, \alpha) = B_0(\alpha)[k(\alpha) f_1(x, y, \alpha) + (1 - k(\alpha)) f_2(x, y, \alpha)]; \qquad (8)$$

where k(d) is a parameter, characterizing the optical properties of the planet

$$0 \leq k(\alpha) \leq 1.$$

Let us now study the systematic deviations between the measured and true brightness distribution which satisfies (8). /84

Let us first study the behavior of the Laplacian $\Delta f(x, y, \alpha)$ within the region bounded by the luminous disc of the planet.

According to (5), the value of the Laplacian within this region, with the exception of the edge zone on the order of σ, determines the distortions of the true brighthess distribution over the planet disc.

Close to the equator of intensity, between the subsolar meridian and the terminator, the difference between $f_1(x, y, \alpha)$ and $f_2(x, y, \alpha)$ is small when α is not too close to zero. In the region between the limb and the subsolar meridian $f_2(x, y, \alpha)$ equals a constant value. Therefore, to study the behavior of $\Delta f(x, y, \alpha)$ for any fixed α it is sufficient to know $\Delta f_1(x, y, \alpha)$. Knowing $\Delta f_1(x, y, \alpha)$, we may find the distortions in the brightness distribution over the disc (8) for any fixed k and α. For this purpose, it is sufficient to assume that the value of k in the region between the subsolar meridian and the terminator equals 1, and between the limb and the subsolar meridian the distortions calculated for $f_1(x, y, \alpha)$, must be multiplied by k. Therefore, all of the following discussion will be carried out for the Lambert reflection law. Then

$$\Delta (f_1(x, y, \alpha) = -\cos\alpha \frac{[2-(x^2+y^2)]}{[1-(x^2+y^2)]^{3/2}},$$

and the measured brightness distribution, according to (5), assumes the form

$$F(x, y, \alpha) = x\sin\alpha + \cos\alpha\sqrt{1-(x^2+y^2)} - \frac{\sigma^2}{2}\cos\alpha\frac{[2-(x^2+y^2)]}{[1-(x^2+y^2)]^{3/2}}.$$

It may be readily seen that within this region $\Delta f_1(x, y, \alpha)$ has one maximum at the point with the coordinates (0,0). The value of the Laplacian, as well as that of the distortions, remains constant for fixed α on any circle with a center coinciding with the center of the disc. The modulus $\Delta f(x, y, \alpha)$ increases with the distance from the center of the disc approximately as

$\cos\alpha \cdot 2\,(1 + r^2)$.

Since the value of the Laplacian at $\alpha < 90°$ is negative, at all points within the illuminated disc the measured brightness will be too low. The maximum understatement of the brightness will be observed at the maximum distance from the center of the disc, i.e., close to the limb. The brightness maximum will be displaced from the subsolar point in the direction toward the center of the disc.

For a more detailed examination of the distortions produced, it is advantageous to use the reciprocity principle.

In planetocentric coordinates, the reciprocal points are arranged symmetrically with respect to the equator of intensity and the meridian passing through the mirror point with the longitude $\lambda = \alpha/2$.

In a rectangular coordinate system with the origin coinciding with the center of the planet disc, three reciprocal points with the following coordinates

$$x_2 = \left(\sin\alpha\sqrt{1-x_1^2} - x_1\cos\alpha\right),\ y_2 = y_1;$$
$$x_3 = x_1,\ y_3 = -y_1;$$
$$x_4 = \left(\sin\alpha\sqrt{1-x_1^2} - x_1\cos\alpha\right),\ y_4 = -y_1.$$

correspond to each point with the coordinates x_1, y_1.

At the moment of the opposition, any four reciprocal points lie on a circle drawn from the center of the disc. Since the Laplacian level lines are concentric circles drawn from the center of the disc, the reciprocal points will have identical brightness /85
values which are too low, and consequently the observed brightness distribution will satisfy the reciprocity principle.

At a phase which does not equal zero, the reciprocal points will not be arranged on the Laplacian level lines, and the distortion of the reciprocal points will be different. It may be readily seen that the brightness of two points lying on the side of the limb will be lower than the brightness of the points symmetrical to them. The greatest difference in the distortion of the symmetrical points will occur on the equator of intensity. At any phase, the central points of the disc will have minimum distortions under the condition that they are located at a distance greater than σ from the terminator.

It is interesting to note that the same phenomena were observed for Venus by M. Minnaert [1] and V. I. Yezerskiy, [2]. Naturally, we cannot explain this only by observation errors, if only for the reason that it is difficult to make any statements about the optical uniformity of the atmosphere of Venus, since there are numerous articles by different authors pointing to the occurrence of spots on the side of the terminator, for example, [15].

Let us now determine the magnitude of the brightness maximum displacement.

Since the brightness maximum is observed on the equator of intensity, let us examine the behavior of the function

$$F(x,\ 0) = x \sin\alpha + \cos\alpha \sqrt{1-x^2} - \frac{\sigma^2}{2} \cos\alpha \frac{2-x^2}{(1-x^2)^{3/2}}.$$

The point of the extremum of this function will correspond to the measured brightness maximum only at phase angles which are somewhat less than 90°. Assuming that the brightness maximum displacement is small, we find

$$\Delta x \approx -\frac{\sigma^2}{2} \frac{\sin\alpha}{\cos^2\alpha} (4-\sin^2\alpha). \quad (9)$$

We cannot use this formula at phase angles greater than 70-80°, since the brightness maximum of the undistorted function lies in the region where the asymptotic formula (5) is not applicable. However, as numerical calculations show, formula (9) is the correct concept regarding the brightness maximum displacement. The brightness maximum at any phase will be displaced toward the terminator. The maximum displacement will occur at the phase angle 90°, and at the time of the opposition the brightness maximum will be observed at the center of the disc. At phase angles greater than 90°, the maximum displacement will decrease.

Let us now determine the magnitude of the distortions of the edge zone of the planet.

At $\alpha = 0$, the function $f(x, y, 0)$ has the widest spectrum in the case when $k(\alpha) = 0$, i.e., $f(x, y, 0) = f_2(x, y, 0) = 1$ over the entire disc.

Since the value of the distortions is determined by the limitation of the higher spatial frequencies, then $f(x, y, 0)$ when k = 0 will be distorted to the greatest extent.

Since $\Delta f_2(x, y, 0)$ equals zero, according to (5) there will be practically no distortion over the entire disc, with the exception of the region with a width on the order of σ from the edge of the disc (more precisely, at a distance of 3σ the magnitude of the distortions will not exceed 0.2%). In this case, all of the distortions may lie outside of the region of applicability of the asymptotic expression (5). In this case, the measured brightness distribution F(x,y) may be represented in the form

$$F(x, y, 0) = \frac{1}{2\pi\sigma^2} \int_{-1}^{+1} dx' \int_{-\sqrt{1-x'^2}}^{\sqrt{1-x'^2}} e^{-\frac{(x-x')^2+(y-y')^2}{2\sigma^2}} dy'.$$

Changing to the polar coordinates, we find /86

$$F(\rho)=\frac{1}{\sigma^2}\int\limits_0^1 e^{-\frac{\rho^2-\rho'^2}{2\sigma^2}} I_0\left(\frac{\rho\rho'}{\sigma^2}\right)\rho' d\rho'.$$

Here $I_0\left(\frac{\rho\rho'}{\sigma^2}\right)$ is the Bessel function of the imaginary argument.

At ρ which are close to unity, in view of the smallness of σ, let us use the asymptotic expansion $I\left(\frac{\rho\rho'}{\sigma^2}\right)$ at $\left(\frac{\rho\rho'}{\sigma^2}\right)\to\infty$.

Disregarding terms on the order of σ^3, we find

$$I_0\left(\frac{\rho\rho'}{\sigma^2}\right)\approx e^{\frac{\rho\rho'}{\sigma^2}}\cdot\frac{\sigma}{\sqrt{2\pi\rho\rho'}},$$

$$F(\rho)=\frac{1}{\sqrt{2\pi\sigma^2\rho}}\int\limits_0^1 e^{-\frac{(\rho-\rho')^2}{2\sigma^2}}\sqrt{\rho'}d\rho'. \qquad (10)$$

It may be readily seen that this formula describes the behavior of the measured brightness distribution close to the limb at k = 0 for any $\alpha<90°$.

For an arbitrary k, formula (10) may be regarded as the expression giving the upper boundary of the distortions close to the limb at $\alpha<90°$.

With respect to the terminator, for the equator of intensity we may use the one-dimensional approximation with a great degree of accuracy, since with an increasing α there is simultaneously a decrease in the curvature of the terminator projection on the mapped plane and the second derivative $f(x, y, \alpha)$, passing through 0 when $\alpha=90°$.

For narrow crescents, i.e., for phase angles $\alpha\gg 90°$, it is most advantageous to determine the distortions by using (3).

Formula (5) was obtained under the assumption of symmetry and smallness of the region in which the kernel of the integral equation (1) was specified. The kernel form determines the coefficient in front of the Laplacian in (5).

If the distortions are determined by diffraction at the input diaphragm of the telescope, the kernel of (1) has the form

$$g(r) = \left[\frac{I_1\left(\frac{\pi D}{r_0} r\right)}{\frac{\pi D}{r_0} r}\right]^2,$$

where D — is the lens diameter,

r_0 — radius of the first diffraction circle.

To determine the distortions g(r) may be represented in the form

$$g(r) \approx e^{-r^2/2\sigma^2} \text{ c } \sigma \approx 0.4 r_0.$$

Thus, a deterioration in the atmospheric conditions may in the first approximation be regarded as a certain equivalent decrease in the telescope diameter.

Under ideal atmospheric conditions, for a telescope with a 30 cm diameter, the radius of the first diffraction ring equals 0".5 and σ = 0".2 corresponding to it.

Let us now determine the distortions in the brightness distribution over the disc of Mars for the opposition in 1967. At the time of the opposition, the diameter of Mars equalled 15".5, and at a maximum phase angle $\alpha = 43°49' - 8"$. Under ideal atmospheric conditions, during an opposition for a 30 cm telescope $\sigma \approx 0{,}025$, and at $\alpha \approx 45° - \sigma \approx 0{,}05$. We give a table below for the distortions of the brightness distribution over the disc of Mars for different σ.

$\alpha = 0°$

σ	r_0''	Br. understmt. in disc center, %	Br. understmt, dis. of 0.87 from disc center, %
0,025 *	0",5	0,07	0,33
0,05	1"	0,25	1,25
0,075	1",5	0,56	2,75
0,1	2"	1,00	4,9

$\alpha = 45°$

σ	r_0''	Recip. dist. $x_1 = 0,87$; $x_2 = -0,26$	Br. max displ. to limb, $\Delta\lambda$
0,05	0",5	0,85%	30′
0,1	1",0	3,4%	2°
0,15	1",5	7,65%	4°20′

* Translator's note: Commas in numbers represent decimal points.

In spite of the almost identical accuracy of individual photometric measurements during the last decade, random errors in planetary photometry have been greatly reduced, due to the great amount of measurements which can be made by present day equipment.

In our opinion, in recent times, the accuracy of photometry may have been primarily limited by systematic errors such as we have discussed. If there is no correction, then these errors must be determined in each specific case, particularly since, from the point of view of experiments (we are referring to photoelectric measurements of the brightness distribution) it is very simple to measure the brightness distribution over the stellar image.

REFERENCES

1. Minnaert, M. Bull. of the Astr. Inst. of the Netherlands, Vol. 10, No. 367, 1946.

2. Yezerskiy, V. I. Photographic Photometry of Venus with Light Filters. Trudy KhAO, 12. Izdatel'stvo Khar'kovskogo gosudarstvennyy universiteta im. A.M. Gor'kogo (KhGU) 1957.

3. Koval', I. K. Astron. Tsirkulyar, Vol. 1, No. 319, 1965.

4. Bugayenko, L. A., O. I. Bugayenko, I. K. Koval' and A. V. Morozhenko. Electrophotometric Cross Sections of the Disc of Mars in the 355-600 m Spectral Interval. Collection: Fizika luny i planet (Physics of the Moon and Planets). Izdatel'stvo Naukova Dumka, Kiev, 1966.

5. Barabashov, N. P., V. I. Garazha, and V. N. Dudinov. Considerations on the Possibility of Curvature of Planetary Photometric Cross Sections. Astronomicheskiy Zhurnal, Vol. XLIII, Vol. 1, 1966, pp. 144-148.

6. Barabashov, N. P., Yu. V. Aleksandrov and V. I. Garazha. In this collection.

7. Struv'ye, G. Izvestiya Peterburgskoy Akademii Nauk, Vol. 7, No. 8, 1882, p. 30.

8. Fesenkov, V. G. Russkiy Astron. Zhurnal, Vol. 25, 1925, p. 171.

9. Barabashov, N. P. Z. Astrophys, Vol. 7, No. 4, 1933, p. 303.

10. Parshin, I. A. Uchenyy Zapiski LGU, No. 136, 1950, pp. 215-224.

11. Lebedinets, V. N. Absolute Photometric Photography of Jupiter and Saturn with Light Filters. Trudy KhAO, No. 12, Izdatel'stvo KhGU, 1957

12. Akimov, L. A. Reflection of Light from the Lunar Surface. Tsirkulyar KhAO, No. 26, 1963.

-3. Akimov, L. A. O zakone otrazheniya sveta lunnoy poverkhnost'yu (Law of Light Reflection from the Lunar Surface). Author's Abstract of Dissertation. Khar'kov, 1965.

14. Shilov, G. Ye. Matematicheskiy analiz. Spetsial'nyy kurs (Mathematical Analysis. Special Course). Fizmatgiz, 1960.

15. Barabashov, N. N. and I. L. Belkina. Spectrophotometric Studies of the Great Dark Spot on Venus. Vestnik Khar'kovsk. Universiteta, Vol. 2, No. 8, 1965.

16. Koval', I. K. Position of the Brightness Maximum on the Disk of Mars. Collection: Fizika Luny i planet (Physics of the Moon and the Planets). Izdatel'stvo Naukova Dumka, 1966.

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